

An FDTD Multigrid based on Multiresolution Analysis

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Abstract
The principles of multiresolution analysis are applied to derive a novel multigrid scheme for the Finite Difference Time Domain (FDTD) technique. For the first time in literature, a mathematical correct approach for this problem is presented which allows for a priori estimation of the error. A dispersion analysis illustrates the benefits of additional wavelets in the FDTD analysis.

I Introduction

The concept of multiresolution analysis has become a matter of fast growing interest in the electromagnetic society. The use of wavelets in the method of moments for the solution of integral equations in frequency domain has been known since the 1993 Antennas and Propagation Symposium in Ann Arbor, MI [1]. Very recent publications have demonstrated that the application of the method of moments directly to Maxwell's equations allows as well for the use of multiresolution analysis in the time domain [2]. In fact, multiresolution time domain (MRTD) schemes based on Battle-Lemarie scaling functions have shown to exhibit highly linear dispersion characteristics. In comparison to conventional FDTD, savings in computation time and memory of one and two orders of magnitude have been reported in [2]. The objective of this paper is to use the same approach in order to derive a FDTD multigrid which allows for an a-priori estimation of the location and accuracy of the multigrid.

It is known that the FDTD equations can be derived by applying the method of moments to Maxwell's equations using pulses as basis functions. Since pulse functions are the scaling functions in the Haar system, the FDTD technique can be considered as a specific MRTD scheme. Based on these ideas, the multigrid for the first resolution level is derived. Dispersion analysis is performed to demonstrate the improvement by adding the multigrid to the regular FDTD. The FDTD and FDTD multigrid schemes presented here have all the E and H components at the same node point as in the current TLM schemes.

II Derivation of the FDTD Multigrid Using the Haar System

As it is known in the literature, the Haar system is generated by a scaling function $\Phi(x)$ and a mother wavelet $\Psi(x)$ given below:

$$\Phi(x) = \begin{cases} 1 & \text{for } 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

and

$$\Psi(x) = \begin{cases} 1 & \text{for } 0 \leq x < \frac{1}{2} \\ -1 & \text{for } \frac{1}{2} \leq x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Fig.1 shows these two generating functions. In the regular FDTD scheme, the electric and magnetic fields are expanded in terms of pulse functions as shown in the equation below:

$$E(x, t) = \sum_{k,m} E_m h_k(t) h_m(x)$$

where E_m is an unknown constant and $h_k(t)$ and $h_m(x)$ are the pulse functions centered at t_k and x_m .

Applying this expansion to the one dimensional wave equation and using the method of moments (Galerkin's technique) we obtain the following discretized equation:

$$\frac{\Delta x}{c\Delta t} (E_{m+1} - E_{m-1}) = E_{m+1} - E_{m-1}$$

which can also be denoted as:

$$D_t E = E_{m+1} - E_{m-1}$$

where D_t is the differential operator in the time domain. This is the regular FDTD discretization scheme with the E and corresponding H components located at the same nodes. To apply multiresolution analysis, the scaling functions and wavelets of the Haar system are both used as shown below :

$$E(x, t) = \sum_{k,m} h_k(t) [E_m^\phi \Phi_m(x) + E_m^\psi \Psi_m(x)]$$

Using Galerkin's method, two discretized equations are obtained for the one dimensional wave equation. Next, a dispersion analysis is performed in order to compare the characteristics of new scheme with that of the traditional FDTD technique in terms of the dispersion errors. A similar approach can be adopted for the 2D and 3D cases to perform the discretization.

III Dispersion Analysis

The dispersion relation of the FDTD and the MRTD scheme is calculated from the solution of the eigenvalue problem after transforming the equations in the frequency domain [3] - [4]. For the 1D wave equation discussed above, the FDTD dispersion relation is shown below:

$$\frac{\Delta x}{c\Delta t} \sin\left(\frac{\Omega}{2}\right) = \sin\left(\frac{X}{2}\right)$$

where

$$\Omega = \omega\Delta t$$

and

$$\chi = \Delta x k_x$$

This equation is also derived when using Yee's cell. In the equations above, Δx and Δt are the space and time steps respectively and k_x is the magnitude of the wave vector. The dispersion relation of the first order resolution MRTD mesh (FDTD multigrid) for the 1D wave is given by the following equation:

$$\frac{\Delta x}{c\Delta t} \sin\left(\frac{\Omega}{2}\right) = 2\sin\left(\frac{\chi}{4}\right)$$

Fig.2 shows plots of the normalized wave vector component χ as a function of the normalized frequency Ω for the ideal case, the FDTD technique and MRTD scheme. From the figure it can be seen that the dispersion curve of the MRTD scheme is much more linear and closer to the ideal than the FDTD scheme.

Similar analysis has been performed for a 2D problem where the wavelets were applied in one direction and only scaling functions in the other direction. As expected, the dispersion relation is the same as that of the FDTD scheme in the direction in which no wavelets are applied while the dispersion curve substantially improves in the direction in which the wavelets are used. Figs. 3 and 4 show plots of χ as a function of Ω for different directions and validate the claims above.

IV Conclusion

For the first time in literature, a mathematically correct approach for a FDTD multigrid has been given. Since FDTD is based on the expansion of the unknown fields in pulse functions, the principles of multiresolution analysis allows for a consistent additional field expansion in terms of Haar wavelets. Due to the additional wavelets, the resulting MRTD scheme exhibits dispersion characteristics with much less dispersion error than the traditional FDTD scheme.

References

- [1] B. Z. Steinberg and Y. Leviatan, "On the use of wavelet expansions in the method of moments", *IEEE Trans. Antennas Propagat.*, Vol. 41, pp. 610-619, May 1993.
- [2] M. Krumpholz and P. Russer, "On the Dispersion in TLM and FDTD", *IEEE Trans. Microwave Theory Tech.*, Vol. 42, No. 7, pp. 1275-1279, July. 1994.
- [3] L. N. Trefethen, "Group Velocity in Finite Difference Schemes", *SIAM Review*, Vol. 24, No. 2, April 1982.
- [4] D. R. Lynch, K. D. Paulsen, "Origin of Vector Parasitics in Numerical Maxwell Solution", *IEEE Trans. Microwave Theory Tech.*, Vol. 39, No. 3, pp. 383-394, March. 1991.

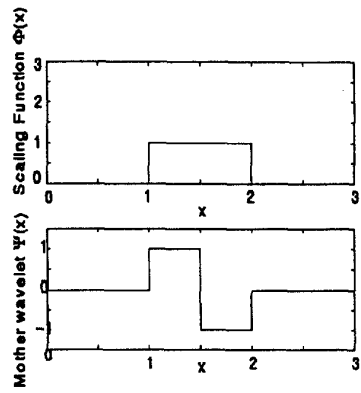


Figure 1: Scaling and Wavelet Functions

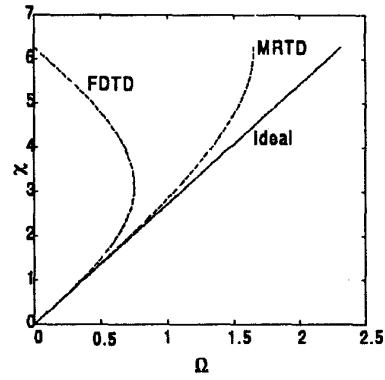


Figure 2: Dispersion Plots for 1D

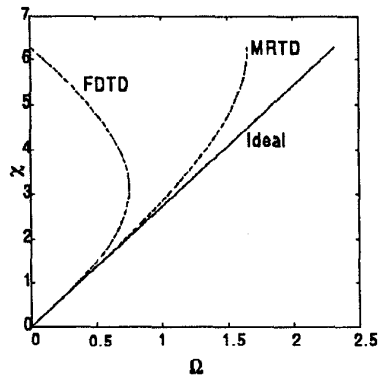


Figure 3: Dispersion Plots for 2D in (1,0,0) direction

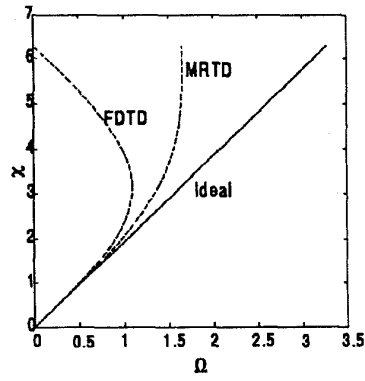


Figure 4: Dispersion Plots for 2D in (1,1,0) direction